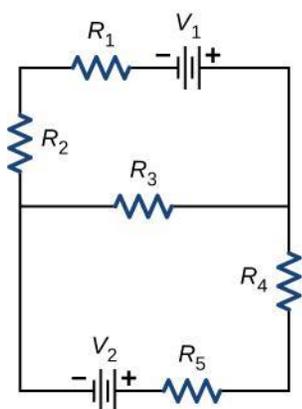


## Kirchhoff's Rules

We have just seen that some circuits may be analyzed by reducing a circuit to a single voltage source and an equivalent resistance. Many complex circuits cannot be analyzed with the series-parallel techniques developed in the preceding sections. In this section, we elaborate on the use of Kirchhoff's rules to analyze more complex circuits. For example, the circuit in the figure below is known as a multi-loop circuit, which consists of junctions. A junction, also known as a node, is a connection of three or more wires. In this circuit, the previous methods cannot be used, because not all the resistors are in clear series or parallel configurations that can be reduced. Give it a try. The resistors  $R_1$  and  $R_2$  are in series and can be reduced to an equivalent resistance. The same is true of resistors  $R_4$  and  $R_5$ . But what do you do then?

Even though this circuit cannot be analyzed using the methods already learned, two circuit analysis rules can be used to analyze any circuit, simple or complex. The rules are known as Kirchhoff's rules, after their inventor Gustav Kirchhoff (1824–1887).



This circuit cannot be reduced to a combination of series and parallel connections. However, we can use Kirchhoff's rules to analyze it.

### Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction:

$$\sum I_{in} = \sum I_{out}$$

- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero:

$$\sum V = 0.$$

We now provide explanations of these two rules, followed by problem-solving hints for applying them and a worked example that uses them.

## Kirchhoff's First Rule

Kirchhoff's first rule (the junction rule) applies to the charge entering and leaving a junction. As stated earlier, a junction, or node, is a connection of three or more wires. Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out.

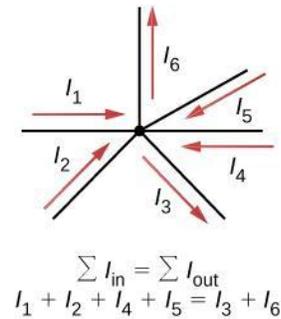


Figure Charge must be conserved, so the sum of currents into a junction must be equal to the sum of currents out of the junction.

Although it is an over-simplification, an analogy can be made with water pipes connected in a plumbing junction. If the wires in the figure above were replaced by water pipes, and the water was assumed to be incompressible, the volume of water flowing into the junction must equal the volume of water flowing out of the junction.

## Kirchhoff's Second Rule

Kirchhoff's second rule (the loop rule) applies to potential differences. The loop rule is stated in terms of potential  $V$  rather than potential energy, but the two are related since  $U = qV$ . In a closed loop, whatever energy is supplied by a voltage source, energy must be transferred into other forms by the devices in the loop since there are no other ways in which energy can be transferred into or out of the circuit. Kirchhoff's loop rule states that the algebraic sum of potential differences, including voltage supplied by the voltage sources and resistive elements, in any loop must be equal to zero. For example, consider a simple loop with no junctions, as in the figure below.

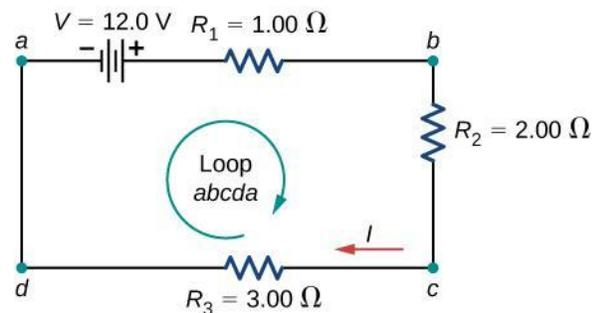


Figure A simple loop with no junctions. Kirchhoff's loop rule states that the algebraic sum of the voltage differences is equal to zero.

The circuit consists of a voltage source and three external load resistors. The labels  $a$ ,  $b$ ,  $c$ , and  $d$  serve as references, and have no other significance. The usefulness of these labels will become apparent soon. The loop is designated as Loop  $abcd$ , and the labels help keep track of the voltage differences as we travel around the circuit. Start at point  $a$  and travel to point  $b$ . The voltage of the voltage source is added to the equation and the potential drop of the resistor  $R_1$  is subtracted. From point  $b$  to  $c$ , the potential drop across  $R_2$  is subtracted. From  $c$  to  $d$ , the potential drop across  $R_3$  is subtracted. From points  $d$  to  $a$ , nothing is done because there are no components.

The figure below shows a graph of the voltage as we travel around the loop. Voltage increases as we cross the battery, whereas voltage decreases as we travel across a resistor. The potential drop, or change in the electric potential, is equal to the current through the resistor times the resistance of the resistor. Since the wires have negligible resistance, the voltage remains constant as we cross the wires connecting the components.

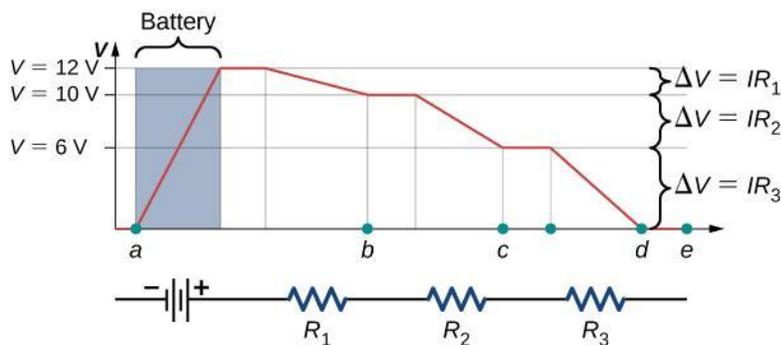


Figure A voltage graph as we travel around the circuit. The voltage increases as we cross the battery and decreases as we cross each resistor. Since the resistance of the wire is quite small, we assume that the voltage remains constant as we cross the wires connecting the components.

Then Kirchhoff's loop rule states

$$V - IR_1 - IR_2 - IR_3 = 0.$$

The loop equation can be used to find the current through the loop:

$$I = \frac{V}{R_1 + R_2 + R_3} = \frac{12.00 \text{ V}}{1.00 \Omega + 2.00 \Omega + 3.00 \Omega} = 2.00 \text{ A}.$$

This loop could have been analyzed using the previous methods, but we will demonstrate the power of Kirchhoff's method in the next section.

### Applying Kirchhoff's Rules

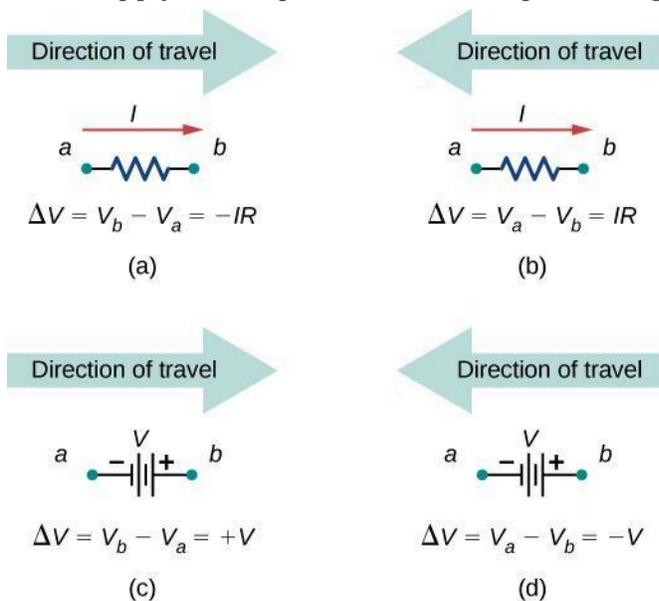
By applying Kirchhoff's rules, we generate a set of linear equations that allow us to find the unknown values in circuits. These may be currents, voltages, or resistances. Each time a rule is applied, it produces an equation. If there are as many independent equations as unknowns, then the problem can be solved.

Using Kirchhoff's method of analysis requires several steps, as listed in the following procedure.

### Problem-Solving Strategy

#### Kirchhoff's Rules

1. Label points in the circuit diagram using lowercase letters  $a, b, c, \dots$ . These labels simply help with orientation.
2. Locate the junctions in the circuit. The junctions are points where three or more wires connect. Label each junction with the currents and directions into and out of it. Make sure at least one current points into the junction and at least one current points out of the junction.
3. Choose the loops in the circuit. Every component must be contained in at least one loop, but a component may be contained in more than one loop.
4. Apply the junction rule. Again, some junctions should not be included in the analysis. You need only use enough nodes to include every current.
5. Apply the loop rule. Use the map in the figure below.



Each of these resistors and voltage sources is traversed from  $a$  to  $b$ . (a) When moving across a resistor in the same direction as the current flow, subtract the potential drop. (b) When moving across a resistor in the opposite direction as the current flow, add the potential drop. (c) When moving across a voltage source from the negative terminal to the positive terminal, add the potential drop. (d) When moving across a voltage source from the positive terminal to the negative terminal, subtract the potential drop.

Let's examine some steps in this procedure more closely. When locating the junctions in the circuit, do not be concerned about the direction of the currents. If the direction of current flow is not obvious, choosing any direction is sufficient as long as at least one current points into the junction and at least one current points out of the junction. If the arrow is in the

opposite direction of the conventional current flow, the result for the current in question will be negative but the answer will still be correct.

The number of nodes depends on the circuit. Each current should be included in a node and thus included in at least one junction equation. Do not include nodes that are not linearly independent, meaning nodes that contain the same information.

Consider the circuit below. There are two junctions in this circuit: Junction *b* and Junction *e*. Points *a*, *c*, *d*, and *f* are not junctions, because a junction must have three or more connections. The equation for Junction *b* is  $I_1 = I_2 + I_3$ , and the equation for Junction *e* is  $I_2 + I_3 = I_1$ . These are equivalent equations, so it is necessary to keep only one of them.

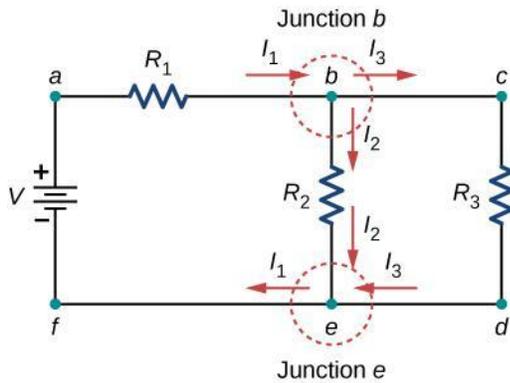
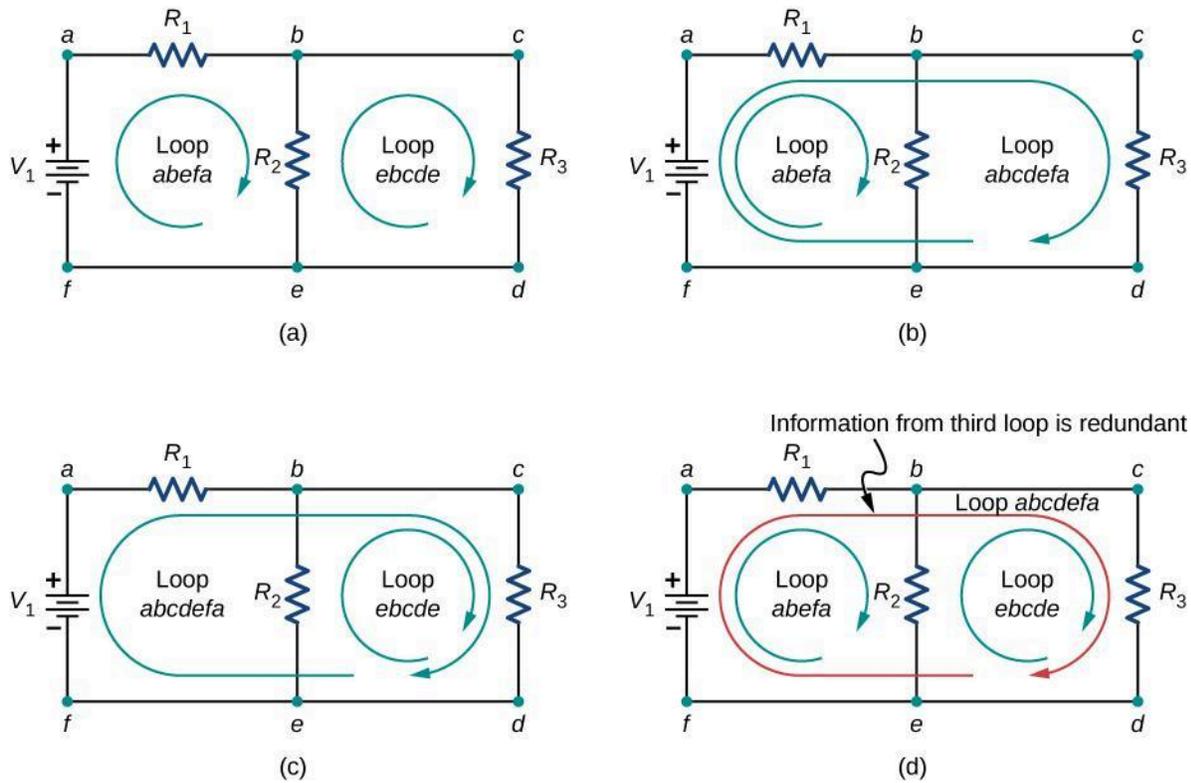


Figure At first glance, this circuit contains two junctions, Junction *b* and Junction *e*, but only one should be considered because their junction equations are equivalent.

When choosing the loops in the circuit, you need enough loops so that each component is covered once, without repeating loops. The figure below shows four choices for loops to solve a sample circuit; choices (a), (b), and (c) have a sufficient amount of loops to solve the circuit completely. Option (d) reflects more loops than necessary to solve the circuit.



Panels (a)–(c) are sufficient for the analysis of the circuit. In each case, the two loops shown contain all the circuit elements necessary to solve the circuit completely. Panel (d) shows three loops used, which is more than necessary. Any two loops in the system will contain all information needed to solve the circuit. Adding the third loop provides redundant information.

Consider the circuit in the figure below (a). Let us analyze this circuit to find the current through each resistor. First, label the circuit as shown in part (b).

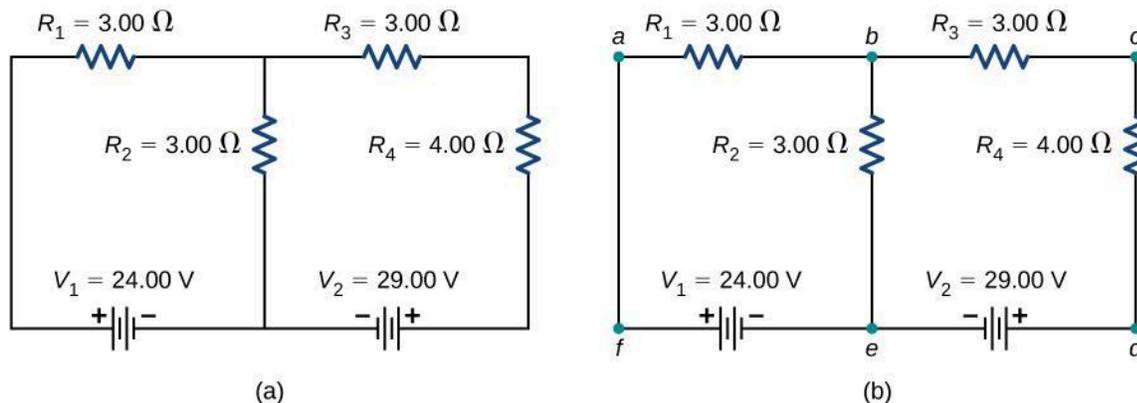
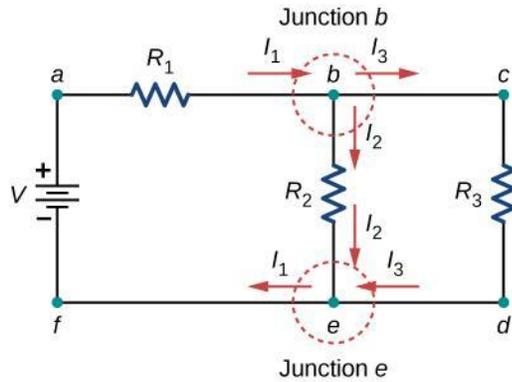


Figure (a) A multi-loop circuit. (b) Label the circuit to help with orientation.

Next, determine the junctions. In this circuit, points  $b$  and  $e$  each have three wires connected, making them junctions. Start to apply Kirchhoff's junction rule ( $\sum I_{in} = \sum I_{out}$ ) by

drawing arrows representing the currents and labeling each arrow, as shown in the figure below(b). Junction  $b$  shows that  $I_1 = I_2 + I_3$  and Junction  $e$  shows that  $I_2 + I_3 = I_1$ . Since Junction  $e$  gives the same information of Junction  $b$ , it can be disregarded. This circuit has three unknowns, so we need three linearly independent equations to analyze it.



- (a) This circuit has two junctions, labeled  $b$  and  $e$ , but only node  $b$  is used in the analysis.  
 (b) Labeled arrows represent the currents into and out of the junctions.

Next we need to choose the loops. In the figure below, Loop  $abefa$  includes the voltage source  $V_1$  and resistors  $R_1$  and  $R_2$ . The loop starts at point  $a$ , then travels through points  $b$ ,  $e$ , and  $f$ , and then back to point  $a$ . The second loop, Loop  $ebcde$ , starts at point  $e$  and includes resistors  $R_2$  and  $R_3$ , and the voltage source  $V_2$ .

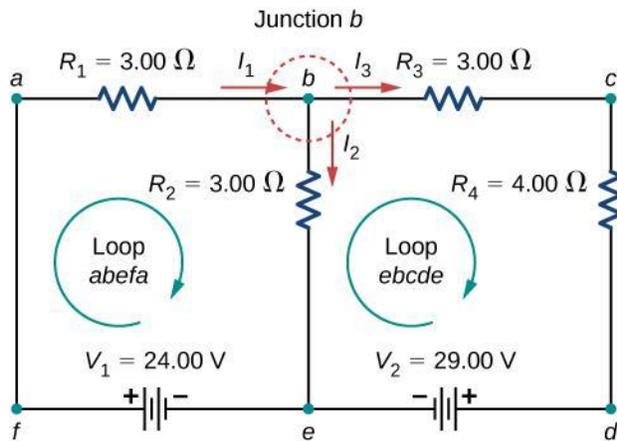


Figure 10.28 Choose the loops in the circuit.

Now we can apply Kirchhoff's loop rule, using the map in above. Starting at point  $a$  and moving to point  $b$ , the resistor  $R_1$  is crossed in the same direction as the current flow  $I_1$ , so the potential drop  $I_1 R_1$  is subtracted. Moving from point  $b$  to point  $e$ , the resistor  $R_2$  is crossed in the same direction as the current flow  $I_2$  so the potential drop  $I_2 R_2$  is subtracted. Moving from point  $e$  to point  $f$ , the voltage source  $V_1$  is crossed from the negative terminal

to the positive terminal, so  $V_1$  is added. There are no components between points  $f$  and  $a$ . The sum of the voltage differences must equal zero:

$$\text{Loop } abefa: -I_1R_1 - I_2R_2 + V_1 = 0 \text{ or } V_1 = I_1R_1 + I_2R_2.$$

Finally, we check loop  $ebcde$ . We start at point  $e$  and move to point  $b$ , crossing  $R_2$  in the opposite direction as the current flow  $I_2$ . The potential drop  $I_2R_2$  is added. Next, we cross  $R_3$  and  $R_4$  in the same direction as the current flow  $I_3$  and subtract the potential drops  $I_3R_3$  and  $I_3R_4$ . Note that the current is the same through resistors  $R_3$  and  $R_4$ , because they are connected in series. Finally, the voltage source is crossed from the positive terminal to the negative terminal, and the voltage source  $V_2$  is subtracted. The sum of these voltage differences equals zero and yields the loop equation

$$\text{Loop } ebcde: I_2R_2 - I_3(R_3 + R_4) - V_2 = 0.$$

We now have three equations, which we can solve for the three unknowns.

$$(1) \text{ Junction } b : I_1 - I_2 - I_3 = 0.$$

$$(2) \text{ Loop } abefa : I_1R_1 + I_2R_2 = V_1.$$

$$(3) \text{ Loop } ebcde : I_2R_2 - I_3(R_3 + R_4) = V_2.$$

To solve the three equations for the three unknown currents, start by eliminating current  $I_2$ . First add Eq. (1) times  $R_2$  to Eq. (2). The result is labeled as Eq. (4):

$$(R_1 + R_2)I_1 - R_2I_3 = V_1.$$

$$(4) 6 \Omega I_1 - 3 \Omega I_3 = 24 \text{ V}.$$

Next, subtract Eq. (3) from Eq. (2). The result is labeled as Eq. (5):

$$I_1R_1 + I_3(R_3 + R_4) = V_1 - V_2.$$

$$(5) 3 \Omega I_1 + 7 \Omega I_3 = -5 \text{ V}.$$

We can solve Eqs. (4) and (5) for current  $I_1$ . Adding seven times Eq. (4) and three times Eq. (5) results in  $51 \Omega I_1 = 153 \text{ V}$ , or  $I_1 = 3.00 \text{ A}$ . Using Eq. (4) results in  $I_3 = -2.00 \text{ A}$ . Finally, Eq. (1) yields  $I_2 = I_1 - I_3 = 5.00 \text{ A}$ . One way to check that the solutions are consistent is to check the power supplied by the voltage sources and the power dissipated by the resistors:

$$P_{\text{in}} = I_1V_1 + I_3V_2 = 130 \text{ W},$$

$$P_{\text{out}} = I_1^2R_1 + I_2^2R_2 + I_3^2R_3 + I_3^2R_4 = 130 \text{ W}.$$

Note that the solution for the current  $I_3$  is negative. This is the correct answer, but suggests that the arrow originally drawn in the junction analysis is the direction opposite of conventional current flow. The power supplied by the second voltage source is 58 W and not  $-58$  W.

### Example

#### Calculating Current by Using Kirchhoff's Rules

Find the currents flowing in the circuit in the figure below.

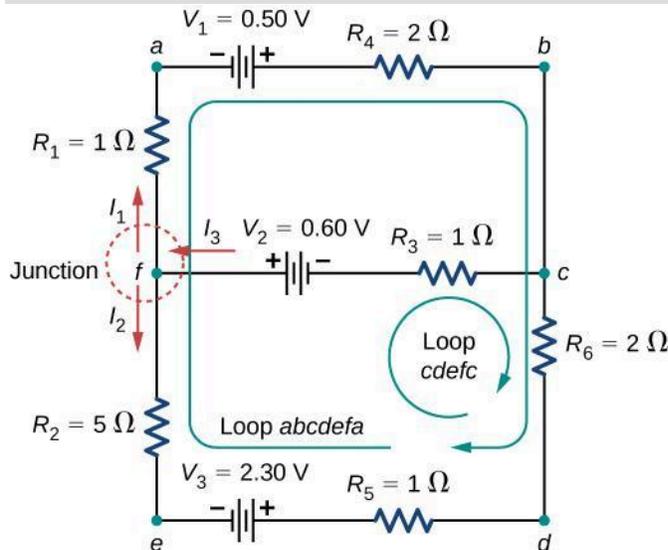


Figure This circuit is combination of series and parallel configurations of resistors and voltage sources. This circuit cannot be analyzed using the techniques discussed in [Electromotive Force](#) but can be analyzed using Kirchhoff's rules.

#### Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled  $I_1, I_2,$  and  $I_3$  in the figure, and assumptions have been made about their directions. Locations on the diagram have been labeled with letters  $a$  through  $h$ . In the solution, we apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

#### Solution

Applying the junction and loop rules yields the following three equations. We have three unknowns, so three equations are required.

$$\text{Junction } c : I_1 + I_2 = I_3.$$

$$\text{Loop } abcdefa : I_1(R_1 + R_4) - I_2(R_2 + R_5 + R_6) = V_1 - V_3.$$

$$\text{Loop } cdefc : I_2(R_2 + R_5 + R_6) + I_3R_3 = V_2 + V_3.$$

Simplify the equations by placing the unknowns on one side of the equations.

$$\text{Junction } c : I_1 + I_2 - I_3 = 0.$$

$$\text{Loop } abcdefa : I_1 (3 \Omega) - I_2 (8 \Omega) = 0.5 \text{ V} - 2.30 \text{ V}.$$

$$\text{Loop } cdefc : I_2 (8 \Omega) + I_3 (1 \Omega) = 0.6 \text{ V} + 2.30 \text{ V}.$$

Simplify the equations. The first loop equation can be simplified by dividing both sides by 3.00. The second loop equation can be simplified by dividing both sides by 6.00.

$$\text{Junction } c : I_1 + I_2 - I_3 = 0.$$

$$\text{Loop } abcdefa : I_1 (3 \Omega) - I_2 (8 \Omega) = -1.8 \text{ V}.$$

$$\text{Loop } cdefc : I_2 (8 \Omega) + I_3 (1 \Omega) = 2.9 \text{ V}.$$

The results are

$$I_1 = 0.20 \text{ A}, I_2 = 0.30 \text{ A}, I_3 = 0.50 \text{ A}.$$

### Significance

A method to check the calculations is to compute the power dissipated by the resistors and the power supplied by the voltage sources:

$$P_{R_1} = I_1^2 R_1 = 0.04 \text{ W}.$$

$$P_{R_2} = I_2^2 R_2 = 0.45 \text{ W}.$$

$$P_{R_3} = I_3^2 R_3 = 0.25 \text{ W}.$$

$$P_{R_4} = I_1^2 R_4 = 0.08 \text{ W}.$$

$$P_{R_5} = I_2^2 R_5 = 0.09 \text{ W}.$$

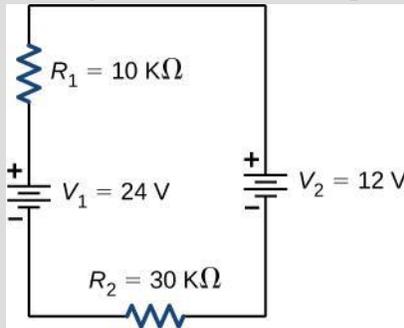
$$P_{R_6} = I_2^2 R_6 = 0.18 \text{ W}.$$

$$P_{\text{dissipated}} = 1.09 \text{ W}.$$

$$P_{\text{source}} = I_1 V_1 + I_2 V_3 + I_3 V_2 = 0.10 \text{ W} + 0.69 \text{ W} + 0.30 \text{ W} = 1.09 \text{ W}.$$

The power supplied equals the power dissipated by the resistors.

**Check Your Understanding** In considering the following schematic and the power supplied and consumed by a circuit, will a voltage source always provide power to the circuit, or can a voltage source consume power?



### Example

#### Calculating Current by Using Kirchhoff's Rules

Find the current flowing in the circuit in the figure below.

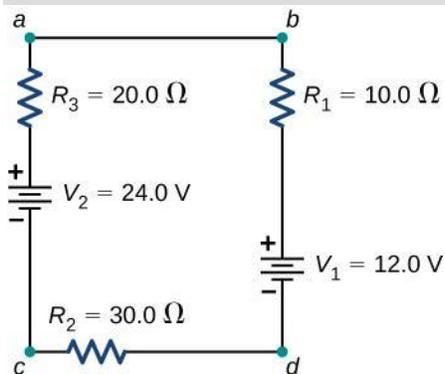


Figure This circuit consists of three resistors and two batteries connected in series. Note that the batteries are connected with opposite polarities.

#### Strategy

This circuit can be analyzed using Kirchhoff's rules. There is only one loop and no nodes. Choose the direction of current flow. For this example, we will use the clockwise direction from point  $a$  to point  $b$ . Consider Loop  $abcda$  and use the figure below to write the loop equation. Note that according to in the figure above, battery  $V_1$  will be added and battery  $V_2$  will be subtracted.

#### Solution

Applying the junction rule yields the following three equations. We have one unknown, so one equation is required:

$$\text{Loop } abcda: \quad -IR_1 - V_1 - IR_2 + V_2 - IR_3 = 0.$$

Simplify the equations by placing the unknowns on one side of the equations. Use the values given in the figure.

$$I(R_1 + R_2 + R_3) = V_2 - V_1.$$

$$I = \frac{V_2 - V_1}{R_1 + R_2 + R_3} = \frac{24 \text{ V} - 12 \text{ V}}{10.0 \Omega + 30.0 \Omega + 10.0 \Omega} = 0.20 \text{ A}.$$

### Significance

The power dissipated or consumed by the circuit equals the power supplied to the circuit, but notice that the current in the battery  $V_1$  is flowing through the battery from the positive terminal to the negative terminal and consumes power.

$$P_{R_1} = I^2 R_1 = 0.40 \text{ W}$$

$$P_{R_2} = I^2 R_2 = 1.20 \text{ W}$$

$$P_{R_3} = I^2 R_3 = 0.80 \text{ W}$$

$$P_{V_1} = IV_1 = 2.40 \text{ W}$$

$$P_{\text{dissipated}} = 4.80 \text{ W}$$

$$P_{\text{source}} = IV_2 = 4.80 \text{ W}$$

The power supplied equals the power dissipated by the resistors and consumed by the battery  $V_1$ .

### Check Your Understanding

When using Kirchhoff's laws, you need to decide which loops to use and the direction of current flow through each loop. In analyzing the circuit above, the direction of current flow was chosen to be clockwise, from point  $a$  to point  $b$ . How would the results change if the direction of the current was chosen to be counterclockwise, from point  $b$  to point  $a$ ?

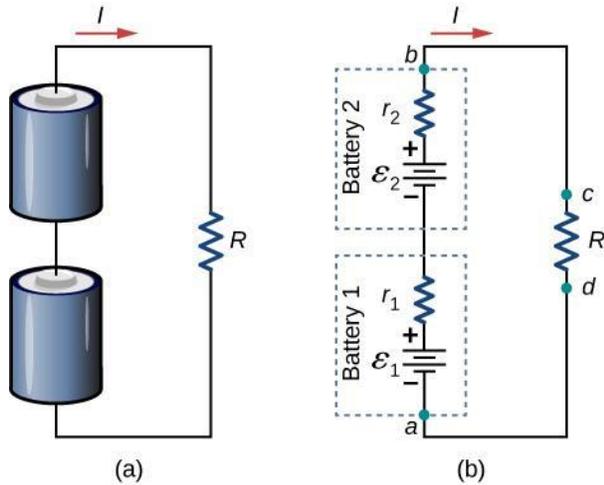
### Multiple Voltage Sources

Many devices require more than one battery. Multiple voltage sources, such as batteries, can be connected in series configurations, parallel configurations, or a combination of the two.

In series, the positive terminal of one battery is connected to the negative terminal of another battery. Any number of voltage sources, including batteries, can be connected in series. Two batteries connected in series are shown below. Using Kirchhoff's loop rule for the circuit in part (b) gives the result

$$\varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2 - IR = 0,$$

$$[(\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)] - IR = 0.$$



(a) Two batteries connected in series with a load resistor. (b) The circuit diagram of the two batteries and the load resistor, with each battery modeled as an idealized emf source and an internal resistance.

When voltage sources are in series, their internal resistances can be added together and their emfs can be added together to get the total values. Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf. In [Figure 10.31](#), the terminal voltage is

$$V_{\text{terminal}} = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) = [(\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)] = (\varepsilon_1 + \varepsilon_2) - Ir_{eq}$$

Note that the same current  $I$  is found in each battery because they are connected in series. The disadvantage of series connections of cells is that their internal resistances are additive.

Batteries are connected in series to increase the voltage supplied to the circuit. For instance, an LED flashlight may have two AAA cell batteries, each with a terminal voltage of 1.5 V, to provide 3.0 V to the flashlight.

Any number of batteries can be connected in series. For  $N$  batteries in series, the terminal voltage is equal to

$$V_{\text{terminal}} = (\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_{N-1} + \varepsilon_N) - I(r_1 + r_2 + \cdots + r_{N-1} + r_N) = \sum_{i=1}^N \varepsilon_i - Ir_{eq}$$

where the equivalent resistance is  $r_{eq} = \sum_{i=1}^N r_i$ .

When a load is placed across voltage sources in series, as in below, we can find the current:

$$(\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2) = IR,$$

$$Ir_1 + Ir_2 + IR = \varepsilon_1 + \varepsilon_2,$$

$$I = \frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R}.$$

As expected, the internal resistances increase the equivalent resistance.

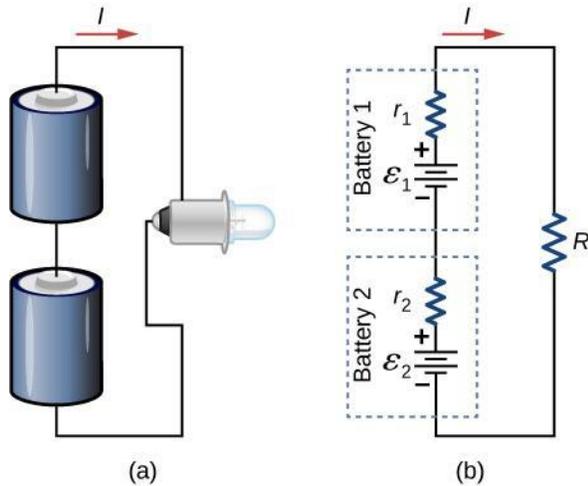


Figure Two batteries connect in series to an LED bulb, as found in a flashlight.

Voltage sources, such as batteries, can also be connected in parallel. The [Figure](#) below shows two batteries with identical emfs in parallel and connected to a load resistance. When the batteries are connect in parallel, the positive terminals are connected together and the negative terminals are connected together, and the load resistance is connected to the positive and negative terminals. Normally, voltage sources in parallel have identical emfs. In this simple case, since the voltage sources are in parallel, the total emf is the same as the individual emfs of each battery.

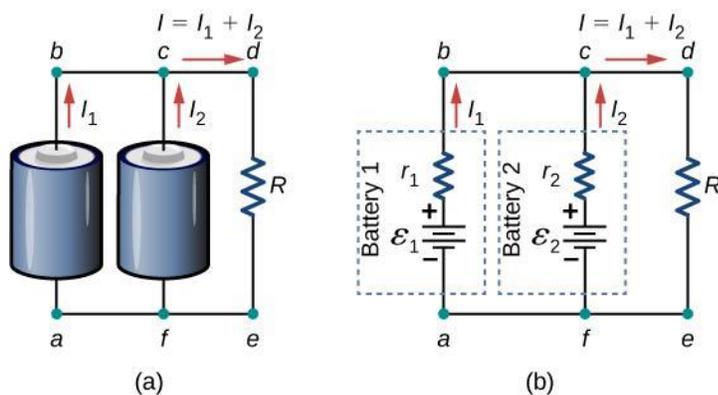


Figure (a) Two batteries connect in parallel to a load resistor. (b) The circuit diagram shows the shows battery as an emf source and an internal resistor. The two emf sources have identical emfs (each labeled by  $\varepsilon$ ) connected in parallel that produce the same emf.

Consider the Kirchhoff analysis of the circuit in the figure above (b). There are two loops and a node at point  $b$  and  $\varepsilon = \varepsilon_1 = \varepsilon_2$ .

Node  $b$ :  $I_1 + I_2 - I = 0$ .

$$\varepsilon - I_1 r_1 + I_2 r_2 - \varepsilon = 0,$$

Loop  $abcfa$ :  $I_1 r_1 = I_2 r_2$ .

$$\varepsilon_2 - I_2 r_2 - IR = 0,$$

Loop  $fcdef$ :  $\varepsilon - I_2 r_2 - IR = 0$ .

Solving for the current through the load resistor results in  $I = \frac{\varepsilon}{r_{eq} + R}$ , where

$r_{eq} = \left( \frac{1}{r_1} + \frac{1}{r_2} \right)^{-1}$ . The terminal voltage is equal to the potential drop across the load resistor  $IR = \left( \frac{\varepsilon}{r_{eq} + R} \right)$ . The parallel connection reduces the internal resistance and thus can produce a larger current.

Any number of batteries can be connected in parallel. For  $N$  batteries in parallel, the terminal voltage is equal to

$$V_{terminal} = \varepsilon - I \left( \frac{1}{r_1} + \frac{1}{r_2} + \dots + \frac{1}{r_{N-1}} + \frac{1}{r_N} \right)^{-1} = \varepsilon - I r_{eq}$$

where the equivalent resistance is  $r_{eq} = \left( \sum_{i=1}^N \frac{1}{r_i} \right)^{-1}$ .

As an example, some diesel trucks use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.

In summary, the terminal voltage of batteries in series is equal to the sum of the individual emfs minus the sum of the internal resistances times the current. When batteries are connected in parallel, they usually have equal emfs and the terminal voltage is equal to the emf minus the equivalent internal resistance times the current, where the equivalent internal resistance is smaller than the individual internal resistances. Batteries are connected in series to increase the terminal voltage to the load. Batteries are connected in parallel to increase the current to the load.

### Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation, which is the conversion of sunlight directly into electricity, is based upon the photoelectric effect. Most solar cells are made from pure silicon. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the

amount of sunlight falling on the cell (the incident solar radiation known as the insolation). Under bright noon sunlight, a current per unit area of about  $100 \text{ mA/cm}^2$  of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

Solar cells, like batteries, provide a direct current (dc) voltage. Current from a dc voltage source is unidirectional. Most household appliances need an alternating current (ac) voltage.