

## Coulomb's Law

Experiments with electric charges have shown that if two objects each have an electric charge, then they exert an electric force on each other. The magnitude of the force is linearly proportional to the net charge on each object and inversely proportional to the square of the distance between them. (Interestingly, the force does not depend on the mass of the objects.) The direction of the force vector is along the imaginary line joining the two objects and is dictated by the signs of the charges involved.

Let

- $q_1, q_2$  = the net electric charges of the two objects;
- $\vec{r}_{12}$  = the vector displacement from  $q_1$  to  $q_2$ .

The electric force  $\vec{F}$  on one of the charges is proportional to the magnitude of its charge and the magnitude of the other charge, and is inversely proportional to the square of the distance between them:

$$F \propto \frac{q_1 q_2}{r_{12}^2}.$$

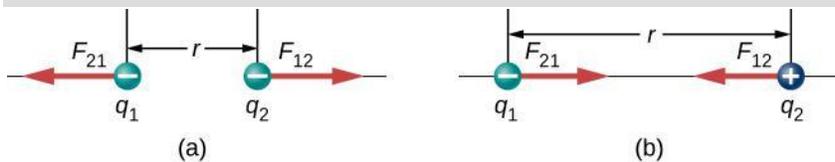
This proportionality becomes an equality with the introduction of a proportionality constant. For reasons that will become clear in a later chapter, the proportionality constant that we use is a collection of constants. (We discuss this constant shortly.)

### Coulomb's Law

The magnitude of the electric force (or Coulomb force) between two electrically charged particles is equal to

$$|F_{12}| = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r_{12}^2}$$

If the charges have the same sign, the force is in the same direction as  $r$  showing a repelling force. If the charges have different signs, the force is in the opposite direction of  $r$  showing an attracting force..



The electrostatic force  $\vec{F}$  between point charges  $q_1$  and  $q_2$  separated by a distance  $r$  is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal

and opposite force) applies as usual—the force on  $q_1$  is equal in magnitude and opposite in direction to the force it exerts on  $q_2$ . (a) Like charges; (b) unlike charges.

It is important to note that the electric force is not constant; it is a function of the separation distance between the two charges. If either the test charge or the source charge (or both) move, then  $r \rightarrow$  changes, and therefore so does the force. An immediate consequence of this is that direct application of Newton's laws with this force can be mathematically difficult, depending on the specific problem at hand. It can (usually) be done, but we almost always look for easier methods of calculating whatever physical quantity we are interested in. (Conservation of energy is the most common choice.)

Finally, the new constant  $\epsilon_0$  in Coulomb's law is called the *permittivity of free space*, or (better) the *permittivity of vacuum*. It has a very important physical meaning that we will discuss in a later chapter; for now, it is simply an empirical proportionality constant. Its numerical value (to three significant figures) turns out to be

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}.$$

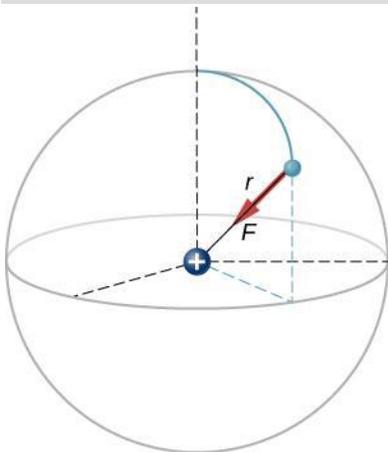
These units are required to give the force in Coulomb's law the correct units of newtons. Note that in Coulomb's law, the permittivity of vacuum is only part of the proportionality constant. For convenience, we often define a Coulomb's constant:

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}.$$

### Example

#### *The Force on the Electron in Hydrogen*

A hydrogen atom consists of a single proton and a single electron. The proton has a charge of  $+e$  and the electron has  $-e$ . In the "ground state" of the atom, the electron orbits the proton at the most probable distance  $5.29 \times 10^{-11} \text{ m}$ . Calculate the electric force on the electron due to the proton.



A schematic depiction of a hydrogen atom, showing the force on the electron. This depiction is only to enable us to calculate the force; the hydrogen atom does not look like this.

### Strategy

For this example, we are treating the electron and proton as two-point particles, each with an electric charge, and we are told the distance between them; we are asked to calculate the force on the electron. We thus use Coulomb's law.

### Solution

Our two charges and the distance between them are,

$$q_1 = +e = +1.602 \times 10^{-19} \text{ C}$$

$$q_2 = -e = -1.602 \times 10^{-19} \text{ C}$$

$$r = 5.29 \times 10^{-11} \text{ m.}$$

The magnitude of the force on the electron is

$$F = \frac{1}{4\pi\epsilon_0} \frac{|e|^2}{r^2} = \frac{1}{4\pi\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} \frac{(1.602 \times 10^{-19} \text{ C})^2}{(5.29 \times 10^{-11} \text{ m})^2} = 8.25 \times 10^{-8} \text{ N.}$$

As for the direction, since the charges on the two particles are opposite, the force is attractive; the force on the electron points radially directly toward the proton, everywhere in the electron's orbit. The force is thus expressed as

$$\vec{F} = (8.25 \times 10^{-8} \text{ N})\hat{r}.$$

### Significance

This is a three-dimensional system, so the electron (and therefore the force on it) can be anywhere in an imaginary spherical shell around the proton. In this "classical" model of the hydrogen atom, the electrostatic force on the electron points in the inward centripetal direction, thus maintaining the electron's orbit. But note that the quantum mechanical model of hydrogen is utterly different.

### Check Your Understanding

What would be different if the electron also had a positive charge?

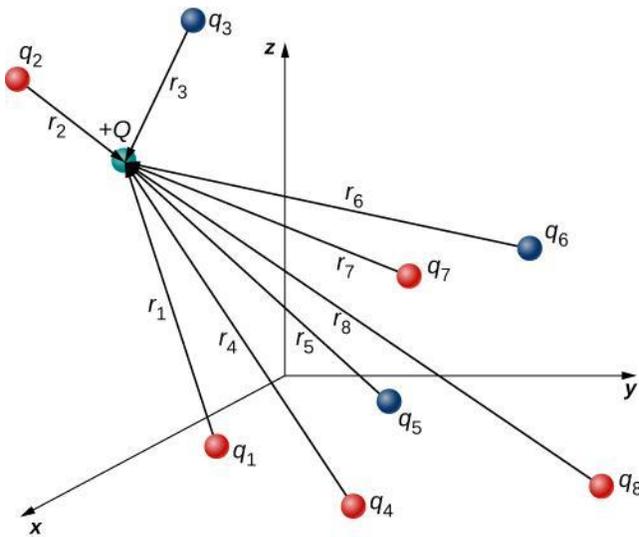
### Multiple Source Charges

The analysis that we have done for two particles can be extended to an arbitrary number of particles; we simply repeat the analysis, two charges at a time. Specifically, we ask the question: Given  $N$  charges (which we refer to as source charge), what is the net electric force that they exert on some other point charge (which we call the test charge)? Note that we use these terms because we can think of the test charge being used to test the strength of the force provided by the source charges.

Like all forces that we have seen up to now, the net electric force on our test charge is simply the vector sum of each electric force exerted on it by each of the individual source charges. Thus, we can calculate the net force on the test charge  $Q$  by calculating the force on it from each source charge, taking one at a time, and then adding all those forces together (as vectors). This ability to simply add up individual forces in this way is referred to as the principle of superposition and is one of the more important features of the electric force. In mathematical form, this becomes

$$\vec{F}(\vec{r}) = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^N \frac{q_i}{r_i^2} \hat{r}_i$$

In this expression,  $Q$  represents the charge of the particle that is experiencing the electric force  $\vec{F}$ , and is located  $\vec{r}$  from the origin; the  $q_i$ 's are the  $N$  source charges, and the vectors  $\hat{r}_i = \vec{r}_i / r_i$  are the displacements from the position of the  $i$ th charge to the position of  $Q$ . Each of the  $N$  unit vectors points directly from its associated source charge toward the test charge. All of this is depicted in the figure below. Please note that there is no physical difference between  $Q$  and  $q_i$ ; the difference in labels is merely to allow clear discussion, with  $Q$  being the charge we are determining the force on.



The eight source charges each apply a force on the single test charge  $Q$ . Each force can be calculated independently of the other seven forces. This is the essence of the superposition principle.

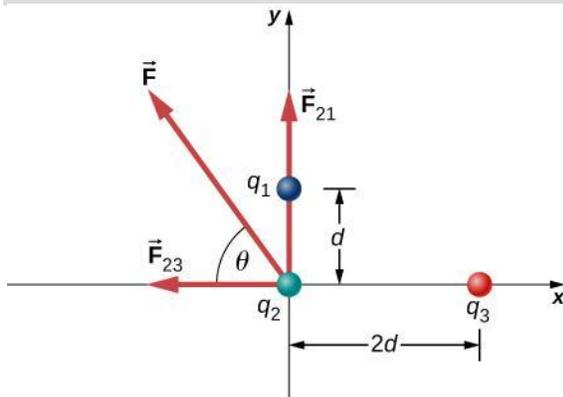
(Note that the force vector  $\vec{F}_i$  does not necessarily point in the same direction as the unit vector  $\hat{r}_i$ ; it may point in the opposite direction,  $-\hat{r}_i$ . The signs of the source charge and test charge determine the direction of the force on the test charge.)

There is a complication, however. Just as the source charges each exerts a force on the test charge, so too (by Newton's third law) does the test charge exert an equal and opposite force on each of the source charges. As a consequence, each source charge would change position. However, the force on the test charge is a function of position; thus, as the positions of the source charge change, the net force on the test charge necessarily changes, which changes the force, which again changes the positions. Thus, the entire mathematical analysis quickly becomes intractable. Later, we will learn techniques for handling this situation, but for now, we make the simplifying assumption that the source charges are fixed in place somehow so that their positions are constant in time. (The test charge is allowed to move.) With this restriction in place, the analysis of charges is known as electrostatics, where "statics" refers to the constant (that is, static) positions of the source charges, and the force is referred to as an electrostatic force.

### Example

#### The Net Force from Two Source Charges

Three different, small charged objects are placed as shown in the figure below. The charges  $q_1$  and  $q_3$  are fixed in place;  $q_2$  is free to move. Given  $q_1 = 2e$ ,  $q_2 = -3e$ , and  $q_3 = -5e$ , and that  $d = 2.0 \times 10^{-7} \text{ m}$ , what is the net force on the middle charge  $q_2$ ?



Source charges  $q_1$  and  $q_3$  each apply a force on  $q_2$ .

#### Strategy

We use Coulomb's law again. The way the question is phrased indicates that  $q_2$  is our test charge, so that  $q_1$  and  $q_3$  are source charges. The principle of superposition says that the force  $q_2$  from each of the other charges is unaffected by the presence of the other charge. Therefore, we write down the force on  $q_2$  from each and add them together as vectors.

#### Solution

We have two source charges ( $q_1$  and  $q_3$ ), a test charge ( $q_2$ ), distances ( $r_{21}$  and  $r_{23}$ ), and we are asked to find a force. This calls for Coulomb's law and superposition of forces. There are two forces:

$$\vec{F} = \vec{F}_{21} + \vec{F}_{23} = \frac{1}{4\pi\epsilon_0} \left[ \frac{q_2 q_1}{r_{21}^2} \hat{j} + \left( -\frac{q_2 q_3}{r_{23}^2} \hat{i} \right) \right].$$

We can't add these forces directly because they don't point in the same direction:  $\vec{F}_{23}$  points only in the  $-x$ -direction, while  $\vec{F}_{21}$  points only in the  $+y$ -direction. The net force is obtained by applying the Pythagorean theorem to its  $x$ - and  $y$ -components:

$$F = \sqrt{F_x^2 + F_y^2}$$

where

$$\begin{aligned} F_x &= -F_{23} = -\frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r_{23}^2} \\ &= -\left( 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \right) \frac{(4.806 \times 10^{-19} \text{ C})(8.01 \times 10^{-19} \text{ C})}{(4.00 \times 10^{-7} \text{ m})^2} \\ &= -2.16 \times 10^{-14} \text{ N} \end{aligned}$$

and

$$\begin{aligned} F_y &= F_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_2 q_1}{r_{21}^2} \\ &= \left( 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \right) \frac{(4.806 \times 10^{-19} \text{ C})(3.204 \times 10^{-19} \text{ C})}{(2.00 \times 10^{-7} \text{ m})^2} \\ &= 3.46 \times 10^{-14} \text{ N}. \end{aligned}$$

We find that

$$F = \sqrt{F_x^2 + F_y^2} = 4.08 \times 10^{-14} \text{ N}$$

at an angle of

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{3.46 \times 10^{-14} \text{ N}}{-2.16 \times 10^{-14} \text{ N}}\right) = -58^\circ,$$

that is,  $58^\circ$  above the  $-x$ -axis, as shown in the diagram.

### Significance

Notice that when we substituted the numerical values of the charges, we did not include the negative sign of either  $q_2$  or  $q_3$ . Recall that negative signs on vector quantities indicate a reversal of the direction of the vector in question. But for electric forces, the direction of the force is determined by the types (signs) of both interacting charges; we determine the force directions by considering whether the signs of the two charges are the same or are opposite. If you also include negative signs from negative charges when you substitute numbers, you run the risk of mathematically reversing the direction of the force you are

calculating. Thus, the safest thing to do is to calculate just the magnitude of the force, using the absolute values of the charges, and determine the directions physically.

It's also worth noting that the only new concept in this example is how to calculate the electric forces; everything else (getting the net force from its components, breaking the forces into their components, finding the direction of the net force) is the same as force problems you have done earlier.

### Check Your Understanding

What would be different if  $q_1$  were negative?